MEC666 — PC1

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The goal of this tutorial is to play with an energy-balance model to understand the concepts of radiative forcing and equilibrium, to identify important physical mechanisms associated to feedback loops and to quantify the sensitivity of the climate system to these processes by applying the concept of equilibrium climate sensitivity.

1 Towards a zero-dimensional climate model

In the first session, the global Earth energy budget for an atmosphere fully absorbing terrestrial emissions is given *at equilibrium* by,

$$\begin{cases} (1-\alpha_{\mathrm{E}})\frac{S}{4} + \sigma T_G^4 & = \sigma T_0^4 \\ \sigma T_0^4 & = 2\sigma T_G^4, \end{cases} \tag{1}$$

where T_0 is the global-mean Earth *surface* temperature at equilibrium, T_G is the global-mean temperature in the Earth *gaseous layer* at equilibrium, $S = 1370\,\mathrm{Wm^{-2}}$ is the solar constant and $\sigma = 5.670\,367 \times 10^{-8}\,\mathrm{Wm^{-2}K^{-4}}$ is the Stefan-Boltzmann constant.

Q1 *Does varying Greenhouse Gas (GHG) concentrations have an impact on the one-layer radiative budget* (1)*?*

In this tutorial, a more general Energy Balance Model (EBM) will be used. The latter is zero-dimensional, in the sense that only global averages are considered as opposed to fields depending on the latitude, the longitude and the altitude. As such, it gives a qualitative representation of some of the dominant mechanisms of the climate system. It lacks the "realism" needed for quantitative studies of the latter, yet is sufficiently physically motivated for pedagogical or theoretical studies. The three state variables, or prognostic variables, of this EBM are the time-dependent global-mean surface temperature T (K), the average CO_2 concentrations (in ppm)[1] and the latitude of the ice-sheet extent ϕ (in degree poleward). The basic diagnostic equations for the incoming radiation Q and the outgoing radiation I, as a function of these prognostic variables, are

$$\begin{cases} Q(T, \phi) & = [1-\alpha_{\mathrm{E}}(\phi)]\,\frac{S}{4} \\ I(T, \mathrm{CO_2}) & = [1-G(T, \mathrm{CO_2})]\,\sigma T^4. \end{cases} \tag{2}$$

[1] The evolution of other GHGs than CO_2 is neglected in this EBM. For quantitative studies, their concentrations should, however, be modeled.

In this model, the global-mean surface temperature T at time t is governed by the following prognostic (ordinary differential) equation,

$$\frac{\mathrm{d}T}{\mathrm{d}t} = (Q - I)/\tau_{\mathrm{RE}}, \tag{3}$$

where $\tau_{\mathrm{RE}} = 100y$ is a constant associated with the heat capacity of the atmosphere and controlling the time scale of relaxation to equilibrium of the temperature.

Q2 *Under which condition is the radiative equilibrium achieved? What does it imply for the evolution of the temperature? For which value of G does the equation* (2) *of the EBM coincide with the radiative budget* (1) *at equilibrium? What do the terms α_{E} and G in* (2) *represent?*

Q3 *Give the formula for the radiative equilibrium temperature T_0 for the EBM* (2)*. Schematically represent T_0 as a function of G for an atmosphere fully transparent to terrestrial radiations to an atmosphere fully absorbing terrestrial radiation, for fixed α_{E} and S. Explain the dependence of T_0 on G.*

Q4 *Based on what you have seen in class and on your physical knowledge, qualitatively describe the most important relationships between the prognostic variables T, CO_2 and ϕ and the diagnostic variables Q, α_{E}, I and G that should be represented in the model. Identify two major feedback loops, i.e. cycles along which a perturbation of one of these variables is propagated through the other variables all the way back to the former. Are these feedbacks positive (amplifying) or negative (damping)?*

2 Computing the climate sensitivity with the model

The IPCC (IPCC, 2014) gives the following definition of the *Equilibrium Climate Sensitivity* (ECS):

Definition 1. *The equilibrium climate sensitivity is the equilibrium change in global and annual mean surface air temperature after doubling the atmospheric concentration of CO_2 relative to pre-industrial levels.*

Our objective is to estimate the ECS from the EBM. The EBM is implemented in Python. It can be run in *integration* mode, for which the prognostic equations are numerically integrated from some initial state to give the transient evolution of the model during a given period. For that purpose, we use the Euler numerical scheme for some time step of integration δt, chosen here as one year. For instance, the temperature for the year t_n is obtained from the previous year t_{n-1} according to

$$T(t_n) = T(t_{n-1}) + \delta t(Q(t) - I(t))/\tau_{\mathrm{RE}}.$$

The model can also be used in *continuation* mode. The latter allows us to directly solve for the stationary points of the model as some continuation variable, or parameter, is changed.

A stationary point is a state for which the prognostic variables do not evolve in time. In the case of the EBM considered here, this means that

$$\begin{cases} \frac{\mathrm{d}T}{\mathrm{d}t}(t) & = 0 \\ \frac{\mathrm{dCO_2}}{\mathrm{d}t}(t) & = 0 \\ \frac{\mathrm{d}\phi}{\mathrm{d}t}(t) & = 0. \end{cases}$$

If this point is (globally) stable, long integrations initialized in its neighborhood eventually converge to it. This process is, however, generally much slower that obtaining the stationary point by continuation and does not work if this point is unstable. The stationary point is instead found by finding the zeros of the vector field in the left-hand side of the system of prognostic equations via the Newton method. This process is repeated for varying values of the continuation variable from an initial state predicted from the previous iteration. For the purpose of this course, you need not know more about these numerical methods.

Q5 *In which mode and how would you use the EBM to compute the ECS? Give the explicit formula that you would apply to compute the ECS according to Definition 1 and assuming that the pre-industrial level is set to the year 1750 with a CO_2 concentration of* 280 ppm.

Open the `pc1.ipynb` notebook in `src/simclimat/example/pc1/`. The latter allows one to run the model in the appropriate mode and to compute the ECS. After running the first cells (see the User Guide for help), you will be offered to switch on (resp. off) changes in the water vapor concentration and in the latitude of the ice caps by checking (resp. unchecking) the check-boxes (see Fig. 1).

Q6 *Ensure that these two processes are active and run the notebook. Describe the relationship between the variables of the model from the resulting figures in light of the physical mechanisms seen in class and of your reply to question* **Q4**.

Q7 *The reference CO_2 concentration value being that of the pre-industrial period, use the results from the figures to discuss the appropriateness of using the ECS to compute the change in the equilibrium surface temperature in response to CO_2 changes for other initial CO_2 concentrations than* 280 ppm *(say for its value of about* 410 ppm *in 2018).*

Q8 *Successively switch off the processes associated with the water vapor and the ice extent and re-run the model. How does the resulting ECS change? Why?*

As reviewed by the IPCC's 5th assessment report, The ECS computed from General Circulation Models (GCMs) are in the range 2.1–4.7 °C. From observations and proxy data, it falls in the range 1.5–4.5 °C.

Q9 *Discuss the results obtained using the EBM in light of these values. What are the major differences between the EBM used here and GCMs participating to the IPCC. Give several reasons motivating the use of complex GCMs instead of such an EBM.*

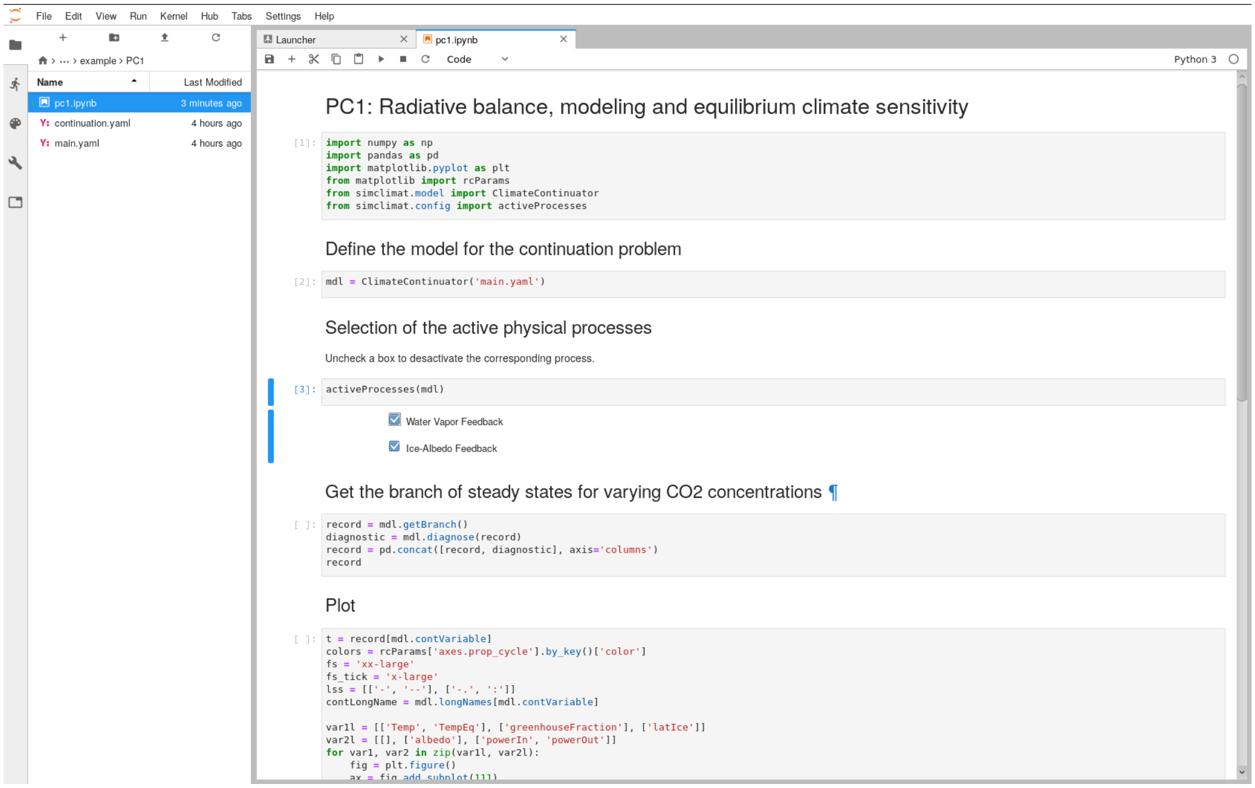


Figure 1: Configuring and running the EBM through the notebook of the PC1.

Some references

- Hartmann, D.L. (1994). *Global Physical Climatology*, Academic Press
- IPCC (2014). *Climate Change 2014: Assessment Report 5 — WGI: The Physical Science Basis*, https://archive.ipcc.ch/report/ar5/wg1/
- IPCC (2018). *Global Warming of* 1.5 °C, https://www.ipcc.ch/sr15/
- IPCC (expected 2022). *Climate Change 2022: Assessment Report 6.*
- Strogatz, S.H. (1994). *Nonlinear Dynamics and Chaos*, Westview.

Take-home message

- Life on Earth and its climate are driven by the energy that the Earth receives from the Sun as shortwave radiation (about half of which is visible).
  - Part of this energy is directly reflected back to space (about 30 %), while most of the rest is absorbed by the surface (oceans, continents, ice) and the troposphere.
  - The absorbed radiation fuels bio-physical processes at the Earth's surface, but is eventually emitted upward as longwave radiation through black-body radiation.

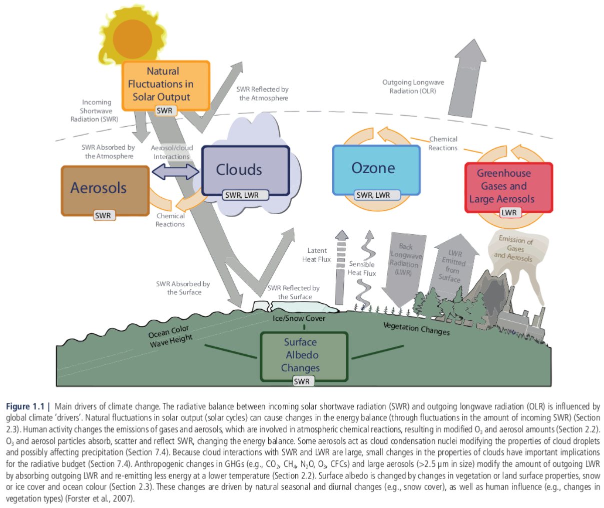
- Less than 10 % of these emissions directly reach space as most of it is absorbed and re-emitted by Greenhouse Gases (GHG) in the troposphere (such as as H_2O, CO_2, CH_4, N_2O, etc.), a phenomenon known as the greenhouse effect.

- At radiative equilibrium, as much energy is returned to space than is received, on average, but an increase in GHG concentrations leads to a temporary imbalance in the radiative forcing with less outgoing than incoming radiations, leading to global warming.
  - The excess energy is primarily absorbed by oceans (about 93 %). Melting ice and warming continents each account for 3 %. Warming the atmosphere makes up the remaining 1 %, yet has a dramatic impact on our climate.
  - However, for a fixed GHG concentration, the only way for the Earth to return to radiative equilibrium is for its surface to warm up to increase black-body radiation until the balance is reestablished.
- While the greenhouse effect involves well-known fundamental physical processes, the response of Earth's climate depends on complex internal feedbacks and is heterogenous in space and time.
  - Observational and modeling climate studies allow us to better understand and quantify the feedbacks controling the Earth's response to radiative forcing.
  - The main positive (amplifying) feedbacks are the (combined) water vapor-lapse rate, the ice-albedo and the cloud feedbacks.
  - The dominant negative (diminishing) feedback is the increased emission of energy through longwave radiation as surface temperature increases.
  - The net feedback is positive.
- The Equilibrium Climate Sensitivity (ECS) is a useful metric summarising the global climate system's temperature response to radiative forcing at equilibrium (after stabilisation).
  - The ECS can be estimated from multiple lines of evidence (observations, models, feedback analysis, etc.) all with their limitations.
  - It is likely in the range 1.5 °C to 4.5 °C.

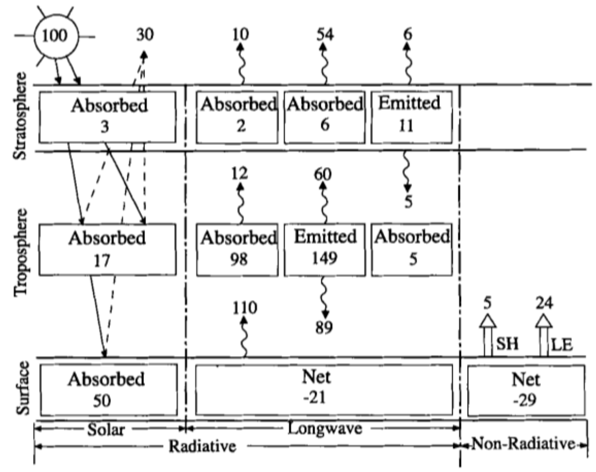
Correction

Q1 The radiative budget (1) is for an atmosphere fully absorbing long-wave radiations emitted by the Earth surface. This means that none of the radiation σT^4 emitted by the Earth is able to directly escape to space. Instead, radiations reaching space all stem from the black body radiation σT_G^4 emitted by the gaseous layer (and from term $\alpha_E S/4$ of the incoming solar radiation being directly reflected). Thus, varying GHG concentrations does not have an effect in this model.

This is thus an extreme case of greenhouse effect. In reality, the atmosphere is partially transparent to terrestrial emissions. See Figure 2a. Yet, less than 90% of the terrestrial emissions today is directly transmitted to space. See Figure 2b.



(a) Source: Chap. 1 in IPCC (2014)



(b) Radiative and nonradiative energy flow diagram for Earth and its atmosphere. Units are percentages of the global-mean insolation (100 units = $342\,\mathrm{Wm}^{-2}$). Source: Hartmann (1994).

Q2 The radiative equilibrium is achieved when the incoming solar radiation is balanced by the outgoing terrestrial radiation. That is when $Q = I$. It follows that the time derivative of the temperature T in (3) is zero, so that the surface temperature remains constant in time[2]. From (1) and (2), one can see that the two models coincide at equilibrium for $G = 0.5$. For lower values of G, part of the terrestrial emissions are able to directly escape to space without being absorbed by the atmosphere. The variable G, let us call it the *greenhouse fraction*, thus controls the intensity of the greenhouse effect. It ranges from a value of 0 for a fully transparent atmosphere to terrestrial radiation to a value of 0.5 for a fully absorbing atmosphere. The variable α_{E} is the same in both models and is the planetary albedo of the Earth. Two major processes controlling the albedo are the cloud cover and the ice-caps extent. Only variations of the latter are taken into account in the EBM considered here.

Q3 The radiative equilibrium temperature is the temperature for which $Q = I$. Thus, from (2), one has that

$$T_0 = \left(\frac{(1 - \alpha_{\mathrm{E}}) S/4}{(1 - G)\sigma} \right)^{1/4}. \tag{4}$$

[2] Note, however, that this is true in the EBM only. In reality, instabilities such as the baroclinic instability, are responsible for the chaotic evolution of the atmosphere that we observe. In this case, invoking ergodicity, the radiative equilibrium is defined as the equality of *long-time averages* of the incoming and outgoing radiation.

The equilibrium temperature is represented as a function of the greenhouse fraction in Figure 3 for an albedo fixed to 0.3 and a solar constant fixed to 1370[3]. The equilibrium temperature is monotonously increasing with the greenhouse fraction. This is expected from the fact that, as the greenhouse effect intensifies so that less outgoing radiation is able to escape to space, the radiative balance is established by increasing the surface temperature, thus increasing the amount of radiation emitted by the Earth.

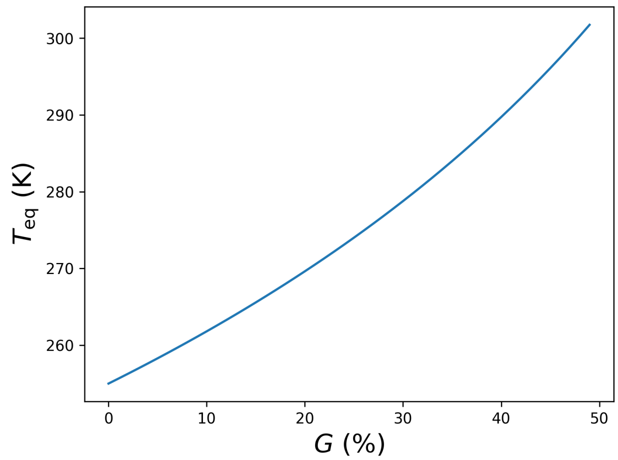


Figure 3: Equilibrium temperature T_0 as a function of the greenhouse fraction G, as given by (4).

Q4 The incoming and outgoing radiations depend on the albedo α_{E} and the greenhouse fraction G, respectively. In this EBM, no variable represents changes in the cloud cover so that, in this model, the albedo only depends on the latitudinal extent of ice-caps. A surface covered by ice has a much higher albedo — from 0.5 for bare ice to 0.9 for ice covered with snow — than the darker ocean — with an albedo of less than 0.1. The latitude ice-caps extent in turn depends on the temperature of the planet. The warmer the planet, the less the ice extent. Thus, a positive perturbation of the temperature results in a decrease of the ice extent, implying an decrease of the albedo, leading to more solar radiation being absorbed by the surface and thus to a higher global surface temperature. This cycle thus has an *amplifying* effect on temperature perturbations. It is referred to as the *ice-albedo feedback*.

Changes in the greenhouse fraction are determined by both CO2 and water vapor concentrations in the atmosphere. The latter depends on the temperature through the thermodynamic law of Clausius-Clapeyron (the exact form of which is not expected to be known by the students)[4]. An increase in temperature leads thus to an increase in humidity. The water vapor being the main greenhouse gas, this increase in humidity leads to an increase

[3] This plot was produced numerically for the purpose of the correction. Only a schematic is expected from the students.

[4] For terrestrial conditions, a 1% change in temperature, about 3 °C is associated with about 20% change in saturation specific humidity (Hartmann, 1994).

in the greenhouse fraction and thus in the temperature. This is a second cycle *amplifying* initial temperature perturbations. It is referred to as the *water-vapor feedback*.

Finally, the increase in the emitted terrestrial radiation I with temperature limits the radiative forcing, and thus constitutes a *negative* feedback, sometimes referred to as the *blackbody radiation* or *Planck* feedback (in reference to the Planck law of black body radiation from which the Stefan-Boltzmann law may be derived).

Q5 To compute the climate sensitivity, only temperatures at the radiative equilibrium are needed. We thus need to compute the stationary state $(T_0(\mathrm{CO_2}), \phi_0(\mathrm{CO_2}))$ corresponding to the radiative equilibrium for a given CO_2 concentration. Assuming that the latter is stable, one solution would be to perform two long simulations; one for a CO_2 concentration fixed to 280 ppm (the pre-industrial level); the other for a CO_2 concentration fixed to $2 \times 280 = 560$ ppm (the CO_2 doubling). The simulations should, however, be sufficiently long for the state of the model to converge sufficiently close to the stationary state. Instead, long integrations can be avoided by directly solving for stationary points via continuation. The continuation may thus be used to compute the temperature $T_0(\mathrm{CO_2})$ at radiative equilibrium for varying CO_2 concentrations. The explicit formula for the ECS corresponding to Definition 1 is then

$$T_0(560\,\mathrm{ppm}) - T_0(280\,\mathrm{ppm}). \tag{5}$$

Q6 The figures represent the evolution of the stationary state of the EBM as CO_2 concentration is increased. The greenhouse fraction increases with CO_2 concentration (middle panel), by definition. As a consequence, temperature increases with CO_2 concentration (top panel), as expected from the fact that CO_2 is one of the major GHGs. The surface temperature and the equilibrium temperature coincide and so do the incoming and outgoing solar radiations (bottom panel). This is necessary since the stationary state is associated with the radiative equilibrium (cf. **Q2**). The incoming and outgoing radiations also increase with temperature. This can be explained by the decrease in albedo (middle panel) which controls the amount of solar radiation reaching the surface (Eq. (2)). The decrease in the albedo is understood from the poleward retreat of the ice-caps (bottom panel) due to raising temperatures.

Q7 The first panel of the figures obtained from the notebook represents the temperature for the radiative equilibrium as a function of CO_2 concentration. It shows a nonlinear dependence of the temperature on CO_2 concentration. In other words, the derivative of the temperature with respect to CO_2 concentration is not constant, but decreases with CO_2 concentration. The ECS may be thought as a finite version of the derivative of the temperature with respect to CO_2 concentration, for a given value of the CO_2 concentration. If the ECS — the change in temperature resulting from a doubling of CO_2 — concentration would be computed from a value of the CO_2 concentration of, say, 410 ppm, its value would then be smaller than the one computed from 280 ppm. The value of the ECS is thus to give a measure of the response of the global mean temperature to CO_2 concentration changes at equilibrium, *to first order*. It also yields a simple observable that can be used to compare the behavior of different climate models and to confront them to historical records. When using the ECS, one should not, however, undermine the complexity of the nonlinear climate system.

Q8 The values of the ECS obtained from the notebook with different combinations of these processes are reported in Table 1. Deactivating these processes results in a decrease

Water Vapor	on	on	off	off
Ice Albedo	on	off	on	off
ECS (K)	4.0	2.8	1.8	1.5

Table 1: Values of the ECS obtained from the notebook toggling the water-vapor feedback and the ice-albedo feedback.

of the ECS, in agreement with the suppression of the *positive* feedbacks associated with the water-vapor and the ice and albedo. According to these values, the greenhouse effect induced by CO_2 alone is responsible for only $1.5/4.0 \times 100 \approx 38\%$ of the ECS. The ice-albedo feedback yields an additional contribution between $(1.8 - 1.5)/4.0 \times 100 \approx 7.5\%$ and $(4.0 - 2.8)/4.0 \times 100 \approx 30\%$ (depending on the order of activation of the processes). The main contribution, ranging from $(4.0-1.8)/4.0\times 100 \approx 55\%$ and $(2.8-1.5)/4.0\times 100 \approx 33\%$ stems from the water-vapor feedback. Taking into account these feedbacks is thus essential for climate change studies[5].

The value of 4.0 K found for the ECS from the EBM lies in the range of both types of estimations of the ECS. The simulations from this EBM are thus coherent with the state of the physical knowledge of the climate system. This model is, however, not fit for the purpose of climate change studies for three main reasons.

First, the equations of the model are *coarse simplifications* of the physical laws of the climate system. They depend on empirical parameters which have been adjusted for the results of the model to fit with observations. As a result, the dependence of these parameters on other important variables (such as the cloud distribution and the cloud height, the meridional energy transport, etc.) is not taken into account and some major feedback loops retro-acting on the global-mean surface temperature are not resolved.

Second, *climate variability* associated with the chaotic nature of the climate system due to instabilities such as baroclinic instability is not present in the model. However, in addition to anthropogenic climate change, this variability contributes to a large fraction of the climate variations experienced by humankind. Yet, according to the IPCC's Special Report on the impacts of global warming of 1.5 °C (2018), since 2000, the estimated level of human-induced global warming[6] has been equal to the level of observed warming with a likely range of $\pm 20\%$ accounting for uncertainty due to contributions from solar and volcanic activity over the historical period (high confidence).

[5] Other feedbacks not discussed here are also important. Let us mention in particular a feedback associated with the meridional energy transport, the longwave and evaporation feedback, the cloud feedback (one of the major source of uncertainty in climate sensitivity studies) and biogeochemical feedbacks.

[6] Global warming is defined in this report as an increase in combined surface air and sea surface temperatures averaged over the globe and over a 30-year period.

Last, the EBM represents global averages of spatial fields only. It does not provide any information on the *spatial distribution* of changes in temperature, precipitation, sea-level height, and in the occurrence extreme events which may have a large impact on our societies.